Exercise 50

Find the horizontal and vertical asymptotes of each curve. If you have a graphing device, check your work by graphing the curve and estimating the asymptotes.

$$y = \frac{1 + x^4}{x^2 - x^4}$$

Solution

Calculate the limits as $x \to \pm\infty$ to determine the horizontal asymptote. In the second limit, make the substitution, $x = -u$, so that as $x \to -\infty$, $u \to \infty$.

$$\lim_{x\to\infty} \frac{1 + x^4}{x^2 - x^4} = \lim_{x\to\infty} \frac{\frac{1}{x^4} + 1}{\frac{1}{x^2} - 1} = \frac{0 + 1}{0 - 1} = -1$$

$$\begin{aligned}
\lim_{x\to-\infty} \frac{1 + x^4}{x^2 - x^4} &= \lim_{u\to\infty} \frac{1 + (-u)^4}{(-u)^2 - (-u)^4} \\
&= \lim_{u\to\infty} \frac{1 + u^4}{u^2 - u^4} \\
&= \lim_{u\to\infty} \frac{\frac{1}{u^4} + 1}{\frac{1}{u^2} - 1} \\
&= \frac{0 + 1}{0 - 1} \\
&= -1
\end{aligned}$$

Therefore, the horizontal asymptote is $y = -1$. The vertical asymptotes are found by setting what's in the denominator equal to zero and solving for x.

$$x^2 - x^4 = 0$$

$$x^2(1 - x^2) = 0$$

$$x^2(1 + x)(1 - x)$$

$$x = 0 \quad \text{or} \quad x = -1 \quad \text{or} \quad x = 1$$

The function is graphed versus x below with the asymptotes labelled.