Exercise 50

Find the horizontal and vertical asymptotes of each curve. If you have a graphing device, check your work by graphing the curve and estimating the asymptotes.

$$y = \frac{1 + x^4}{x^2 - x^4}$$

Solution

Calculate the limits as $x \to \pm \infty$ to determine the horizontal asymptote. In the second limit, make the substitution, x = -u, so that as $x \to -\infty$, $u \to \infty$.

$$\lim_{x \to \infty} \frac{1+x^4}{x^2 - x^4} = \lim_{x \to \infty} \frac{\frac{1}{x^4} + 1}{\frac{1}{x^2} - 1} = \frac{0+1}{0-1} = -1$$
$$\lim_{x \to -\infty} \frac{1+x^4}{x^2 - x^4} = \lim_{u \to \infty} \frac{1+(-u)^4}{(-u)^2 - (-u)^4}$$
$$= \lim_{u \to \infty} \frac{1+u^4}{u^2 - u^4}$$
$$= \lim_{u \to \infty} \frac{\frac{1}{u^4} + 1}{\frac{1}{u^2} - 1}$$
$$= \frac{0+1}{0-1}$$
$$= -1$$

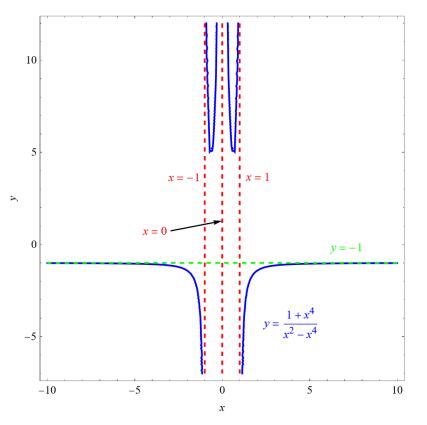
Therefore, the horizontal asymptote is y = -1. The vertical asymptotes are found by setting what's in the denominator equal to zero and solving for x.

$$x^{2} - x^{4} = 0$$

$$x^{2}(1 - x^{2}) = 0$$

$$x^{2}(1 + x)(1 - x)$$

$$x = 0 \quad \text{or} \quad x = -1 \quad \text{or} \quad x = 1$$



The function is graphed versus x below with the asymptotes labelled.